Thermal Expansion along 1-d - Linear Expansion

Consider a 1-dimensional body whose length is l_0 at 0°c. When the body is heated through t°c it expands.

Let l_t be the length of the body at t°c. So, the change in length = $l_t - l_0$

This change in length depends upon two factors.

① It depends upon the initial length of the body.

i.e. $l_t - l_0 \propto l_0$

② It also depends upon the rise in temperature.

i.e. $l_t - l_0 \propto t$

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Combining above two expression, we get

$l_t - l_0 \propto l_0 t$

$\Rightarrow l_t - l_0 = \alpha l_0 t$

Where α is the constant of proportionality and is known as co-efficient of linear expansion.

$\Rightarrow l_t = l_0 + \alpha l_0 t$

$\Rightarrow \boxed{l_t = l_0(1+\alpha t)}$

$\Rightarrow \boxed{\alpha = \frac{l_t - l_0}{l_0 t}}$

If $l_0 = 1$ unit

$t = 1°c$

then $\alpha = l_t - l_0$

So, co-efficient of linear expansion is defined as the change in length per unit length at 0°c per degree centigrade rise in temperature.

→ Thermal expansion along 2-D - Superficial Expansion.

Consider a 2 dimensional body whose area is A_0 at 0°c. When the body is heated through t°c it expands

Let A_t be the area of the body at $t°C$. So the change in area $= A_t - A_0$

This change in ~~length~~ due area depends upon two factors.

① It depends upon the initial area of the body.

i.e. $A_t - A_0 \propto A_0$

② It depends upon the rise in temperature

i.e. $A_t - A_0 \propto t$

Combining above two expression, we get

$A_t - A_0 \propto A_0 t$

$\Rightarrow A_t - A_0 = \beta A_0 t$

where β is the constant of proportionality and is known as co-efficient of superficial expansion.

$\Rightarrow A_t = A_0 + \beta A_0 t$

$\Rightarrow \boxed{A_t = A_0(1+\beta t)}$

$\Rightarrow \beta = \dfrac{A_t - A_0}{A_0 t}$

If $A_0 = 1$ unit

$t = 1°C$

then $\beta = A_t - A_0$

So, co-efficient of superficial expansion is defind as the change in area per unit area at 0°c per degree centigrade rise in temperature.

→ Thermal expansion along 3D - Cubical Expansion:

Consider a 3 dimensional body whose volume is V_0 at 0°c. When the body is heated through t°c it expands.

Let V_t be the volume of the body at t°c.

So, the change in volume = $V_t - V_0$

This change in volume depends upon two factors.

(i) It depends upon the initial volume of the body

i.e. $V_t - V_0 \propto V_0$

(ii) It depends upon the rise in temperature

i.e. $V_t - V_0 \propto t$

Combining above two expression, we get

$V_t - V_0 \propto V_0 t$

$\Rightarrow V_t - V_0 = \gamma V_0 t$

where γ is the constant of proportionality and is known as co-efficient of cubical expansion.

DA-30/10/19

$\Rightarrow V_t = V_0 + \gamma V_0 t$

$\Rightarrow \boxed{V_t = V_0(1+\gamma t)}$

$\Rightarrow \boxed{\gamma = \frac{V_t - V_0}{V_0 t}}$

If $V_0 = 1$ unit

$t = 1°C$,

then $\gamma = V_t - V_0$

So, co-efficient of cubical expansion is defined as the change in volume per unit volume at 0°c per degree centigrade rise in temperature.

→ SI. Unit is

→ Examples of thermal expansion :-

(i) In laying down the rail track, a gap is left in between the two consecutive pieces of rails. In summer the rails expand. If this gap is not there the rails may bend, thereby causing derailment of trains.

(ii) When we want to open a tight jar, it is recommended that we run it under hot water. This expands the lid and the jar opens up easily.

(iii) Cracks in the road when the road expand on heating

(iv) Tyre bursts in hot days when filled full of air due to thermal expansion.

(v). Chimney of a burning lamp cracks when a drop of water is put over it. Due to the heat of flame, chimney is in expanded state. Drop of water suddenly lowers the temperature, which contract, ~~on a part of it which contract~~ thereby causing a crack.

* **Relation among expansion co-efficient:-**

(1) **Relation between α & β :-**

Consider a 2D body whose length is l_0 and breadth is b_0 & area is A_0 at $0°C$.

So $A_0 = l_0 b_0$

All heating the body to $t°C$ it expands. Now l_t, b_t & A_t be the new length, breadth and area of the two dimensional body at $t°C$.

Recalling the (1), we get

$l_t = l_0(1+\alpha t)$

& $b_t = b_0(1+\alpha t)$

& $A_t = l_t b_t$

Recalling the (ii) we get.

$A_t = A_0(1+\beta t)$

$\therefore A_t = l_t b_t$

$\Rightarrow l_t b_t = A_t$

$\Rightarrow l_t b_t = A_0(1+\beta t)$

$\Rightarrow l_0(1+\alpha t)\, b_0(1+\alpha t) = l_0 b_0 (1+\beta t)$

$\Rightarrow \cancel{l_0}\cancel{b_0}(1+\alpha t)(1+\alpha t) = \cancel{l_0}\cancel{b_0}(1+\beta t)$

$\Rightarrow (1+\alpha t)^2 = (1+\beta t)$

$\Rightarrow 1+\beta t = 1+\alpha^2 t^2 + 2\alpha t$

(since α is small, so we neglet the term higher order in α)

$\Rightarrow \cancel{1}+\beta t = \cancel{1}+2\alpha t$

$\Rightarrow \beta t = 2\alpha t$

$\Rightarrow \beta = \frac{2\alpha \cancel{t}}{\cancel{t}}$

$\Rightarrow \boxed{\beta = 2\alpha}$

(ii) <u>Relation between α & γ</u> :-

Consider a 3D body whose length is l_0, breadth is b_0, hight is h_0 and volume is V_0 at 0°c.

So, $V_0 = l_0 b_0 h_0$

All heating the body to t°c it expands. Now l_t, b_t, h_t & V_t be the new length, breadth, height and volume of the three dimensional body at t°c.

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Recalling the (i), we get

$l_t = l_0(1+\alpha t)$

$b_t = b_0(1+\alpha t)$

$h_t = h_0(1+\alpha t)$

& $V_t = l_t b_t h_t$

Recalling the (iii), we get

$V_t = V_0(1+\gamma t)$

$\Rightarrow l_t b_t h_t = V_0(1+\gamma t)$

$\Rightarrow V_0(1+\gamma t) = l_t b_t h_t$

$\Rightarrow l_0 b_0 h_0 (1+\gamma t) = l_0(1+\alpha t)\, b_0(1+\alpha t)\, h_0(1+\alpha t)$

$\Rightarrow l_0 b_0 h_0 (1+\gamma t) = l_0 b_0 h_0 (1+\alpha t)^3$

$\Rightarrow 1+\gamma t = (1+\alpha t)^3$ $\qquad (a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$\Rightarrow 1+\gamma t = 1+3\alpha t+3\alpha^2 t^2+\alpha^3 t^3$

(since α is small, so we neglet the term higher order in α)

$\Rightarrow 1+\gamma t = 1+3\alpha t$

$\Rightarrow \gamma = \frac{3\alpha t}{t}$

$\Rightarrow \boxed{\gamma = 3\alpha}$

$\boxed{\alpha = 0.001}$

* $\beta = 2\alpha$

$\gamma = 3\alpha$

$\therefore \alpha : \beta : \gamma$

$= \alpha : 2\alpha : 3\alpha$

$\Rightarrow 1 : 2 : 3$

$\therefore \boxed{\alpha : \beta : \gamma = 1 : 2 : 3}$

* Heat (H) :-

It is a form of thermal energy that give the sensation of hotness and coldness of a body.

Imp S.I. - Unit :- Joule (J)

M.K.S. :- kilo calorie (K cal)

C.G.S :- Calorie (cal)

F.P.S :- BTU (British Thermal Unit)

Dimension :- $[M^1L^2T^{-2}]$

* Temperature (T) :-

It is the degree of hotness and coldness of a body.

S.I. Unit :- °K (Kelvin)

M.K.S. :- °C

C.G.S. :- °C

Dimension :- $[K^1]$

Heat energy always flows from a body at high temperature to a body at low temperature.

* Difference between heat and temperature

Heat	Temperature
(i) It is a form of thermal energy that give the sensation of hotness and coldness of a body.	(i) It is the degree of hotness and coldness of a body.
(ii) SI Unit of heat is ~~Kelvin~~ Joule (J)	(ii) SI Unit of temperature is Kelvin.

(ii) Dimension of heat is $[M^1L^2T^{-2}]$ | (iii) Dimension of temperature is $[K^1]$

× Scales of Temperature :

① Centigrade scale / Celsius scale :

It is a scale of temperature having lower fix point (melting point of water) at 0 and upper fix point (boiling point of water) at 100. The scale is equally divided into 100 equal parts.

② Fahrenhite scale :

It is a scale of temperature having lower fix point (~~melting point of water~~ melting point of water) at 32°F and upper fix point (~~boiling point water~~ boiling point of water) at 212. The scale is equally divided into 180 equal parts.

③ Kelvin Scale :-

It is a scale of temperature having lower fix point (melting point of water) at 273 and upper fix point (boiling point of water) at 373. The scale is equally divided into 100 equal parts. It is also known as absolute scale.

(1) Reaumer scale :-

It is a scale of temperature having lower fix point (melting point of water) at 0 and upper fix point (boiling point of water) at 80. The scale is equally divided into 80 equal parts.

* Temperature conversion formula :-

$$\frac{\text{Temperature on one scale} - LFP}{UFP - LFP} = \frac{\text{Temperature 2nd scale} - LFP}{UFP - LFP}$$

• $\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273}{373-273} = \frac{R-0}{80-0}$

Soln

$$\frac{C}{100} = \frac{F-32}{180} = \frac{K-273}{100} = \frac{R}{80}$$

Imp * Specific Heat Capacity :-

Let m be the mass of the substance and H be the amount of heat required and Δt be the rise in temperature.

So, Amount of heat required is directly proportional to the mass of the substance for a constant rise in temperature.

$H \propto m$

The amount of heat required is directly proportional to the rise in temperature for a constant mass of substance.

So, $H \propto \Delta T$

Combing above two expressions

$H \propto m(\Delta T)$

$\Rightarrow \boxed{H = ms\Delta T}$

$\Rightarrow \boxed{S = \frac{H}{m(\Delta T)}}$

where s is the constant of proportionality and is known as specific heat capacity.

If $m = 1$ unit

$\Delta T = 1^\circ c$

then $s = H$

So, specific heat capacity of a substance is defined as the amount of heat required to raise the temperature of unit mass of substance through $1^\circ c$.

→ SI unit of sp S = joule $\frac{kg\,m^2 s^{-2}}{kg\,K} = m^2 s^{-2} K^{-1}$

→ Dimension of S = $[M^1 L^2 T^{-2}]$

→ SI unit of S = $J\,kg^{-1}\,{}^\circ K^{-1}$

M.K.S unit = $K\,cal\,kg^{-1}\,{}^\circ C^{-1}$

C.G.S. unit ÷ cal g^{-1} $°c^{-1}$

Dimension - :

$$[s] = \frac{[H]}{[m][\Delta T]} = \frac{[M^1 L^2 T^{-2}]}{[M^1][K^1]}$$

$$= [M^0 L^2 T^{-2} K^{-1}]$$

Imp * There are two types of specific heat capacity in case of gas.

(i) Specific heat capacity at constant volume:-

It is defined as the amount of heat energy required to raise the temperature of 1 g. of gas through 1°c keeping it's volume constant. It is denoted by c_v.

→ For 1 mole of gas it is called molar specific heat capacity at constant volume, and it is denoted by C_v.

$C_v = Mc_v$ where

M = molecular weight of gas.

(ii) Specific Heat Capacity at Constant Pressure

It is defined as the amount of heat energy required to raise the temperature of 1g of gas through 1°c keeping it's pressure constant. It is denoted by c_p.

→ For 1 mole of gas it is called molar specific heat capacity at constant pressure and it is denoted by C_p.

SI Unit - $J\,mol^{-1}\,K^{-1}$

$C_p > C_v$

$C_p = Mc_p$ where

M = molecular weight of gas.

UNIT-?
Mechanical Equivalents of Heat :- Dt - 05/11/19

Dr. James Prescoft Joule after a series of experiment concluded that there is a equivalence between work and heat.

According to him, "whenever heat is converted into work or work into heat the quantity of energy disappearing in one form is equivalent to the quantity of energy appearing in other.

Let W amount of work results in the production of 'H' quantity of heat.

$W \propto H$

$\Rightarrow$ $W = JH$ where

J is the constant of proportionality and is known as Joule's Mechanical equivalents of heat.

$\Rightarrow$ $J = \frac{W}{H}$

If $H = 1 cal$

then $J = W$

So, Joule's mechanical equivalent of heat is defined as the amount of work required to produce unit quantity of heat (1 cal).

→ S.I. unit of $J = \frac{W}{H}$

$= J/Cal$

→ J value is $4.2 J cal^{-1}$, $4.2J = 1 cal$
J has no
→ Dimension $J = \frac{W}{H}$.

$= Kg$.

Imp * 1st law of thermodynamics :-
Statement -

If some quantity of heat energy supplied to a system is capable of doing some work, then the quantity of heat energy absorbed by the system is equal to the sum of increasing in internal energy and external work done by the system.

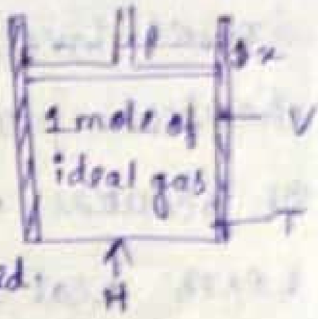
If an infinitesimal amount of heat energy (dH) is supplied to a system and is absorbed by the system, then the corresponding change in internal energy (du) and work done (dw) are also very small.
According to the 1st law of thermodynamics

$dH = du + dw$

this is the mathematically form of 1st law of thermodynamics.

Explanation :

Consider a cylindrical barrel contain of 1 mole of ideal gas whose both side is insulated of both side and bottom is conducted.



Let P be the pressure, V be the volume and T be the temperature produced when H amount of heat applied through bottom.

Let U_1 = initial internal energy
H = Heat energy supplied

At beggining total energy = $U_1 + H$

When H is absorbed by the system then heat energy is converted into work and piston is displaced in x distance.

Let W = work done by the system
U_2 = Final internal energy

At the end, total energy = $U_2 + W$

According to the law of energy conservation

$U_1 + H = U_2 + W$

$\Rightarrow \boxed{H = (U_2 - U_1) + W}$

* Change of State :-

→ It is the physical change of matter.
→ It is a reversible process and it doesn't involve any change of matter's chemical property.
→ It occurs when the substance absorbs/ loses heat energy.

→ Melting :-

It is the process in which solid is converted into a liquid. The temperature at which solid converts to a liquid is know as melting point. Melting point of ice is 0°c.

→ Freezing :

It is the process in which liquid is converted into a solid. ~~The temperature at which liquid converts to a liquid is known as~~ is known as freezing.

→ Vaporisation :

It is the process in which liquid boils and is converted into a gas. The temperature at which liquid boils is known as it's boiling point. Boiling point of water is 100°C

→ **Condensation**:-

It is the process in which gas is converted into a liquid is known as condensation.

→ **Evaporation**:-

The process in which liquid is converted into gas without boiling is known as evaporation.

→ **Sublimation**:-

It is the process in which solid is converted into gas is known as sublimation.

→ **Deposition**:-

It is the process in which gas is converted into solid is known as deposition.

Imp
* **Latent Heat**:-

It is defined as the heat energy in hidden form which is supplied or extracted to change the state of a matter without changing it's temperature.

Let m mass of the substance absorbs 'H' quantity of heat to change it's state at constant temperature, so

Latent heat (L) = $\frac{H}{M}$

$\Rightarrow \boxed{H = mL}$

→ S.I. unit of L $J\,kg^{-1}$

→ M.K.S unit :- $Kcal\,kg^{-1}$

→ C.G.S unit :- $cal\,g^{-1}$

→ It is of 2 types.

(i) <u>Latent heat of fusion</u> (L_f) :-

It is defined as the amount of heat energy supplied per unit mass of the substance at it's melting point to change it's state from solid to liquid without changing it's temperature.

→ Latent heat of fusion for water is 80 $cal\,g^{-1}$.

(ii) <u>Latent heat of vaporisation</u> (L_v) :-

It is defined as the amount of heat energy supplied per unit mass of the substance at it's boiling point to change it's state from liquid to gas without changing it's temperature.

→ Latent heat of vaporisation for water is 540 $cal\,g^{-1}$.

Specific heat	Latent heat
water - 1 $cal g^{-1} °c^{-1}$	L_f = 80 $cal g^{-1}$
ice - 0.5 $cal g^{-1} °c^{-1}$	L_v = 540 $cal g^{-1}$
steam - 0.47 $cal g^{-1} c^{-1}$	

Q Calculate the amount of heat required to convert 5g of ice at -5°c to water at 60°c.

→ mass of ice
m = 5g.

5g of ice at -5°c $\xrightarrow{H_1}$ 5g of ice at 0°c $\xrightarrow{H_2}$ 5g of water at 0°c $\xrightarrow{H_3}$ 5g of water at 60°c

Let H_1 is the amount of heat required to convert 5g of ice at -5°c to ice at 0°c.

Change in temperature (ΔT) = 0 - (-5)°c
= 5°c

$\therefore \Delta T$ = 5°c

Specific heat of ice = 0.5 $cal g^{-1} °c^{-1}$

$\therefore H_1 = m s \Delta T$

$= 5 \times 0.5 \times 5 \; g \times \frac{cal}{g °c} \times °c$

= 12.5 Cal

Let H_2 is the amount of heat required to convert 5g of ice at 0°C to 5g of water at 0°C.

Latent heat of fusion (L_f) = 80 cal g^{-1}

$H_2 = mL_f$

$= 5 \times 80 \, g \frac{cal}{g}$

$= 400$ cal

Let H_3 is the amount of heat required to convert 5g of water at 0°C to 5g of water at 60°C

Change in temperature (ΔT) = 60°C

specific heat of water = 1 cal $g^{-1} °C^{-1}$

$H_3 = ms(\Delta T)$

$= 5 \times 1 \times 60 \, g \frac{cal}{g °C} × °C$

$= 300$ cal.

H is the total amount of heat is required to convert 5g. of ice at -5°C to water at 60°C

$\therefore H = H_1 + H_2 + H_3$

$= (12.5 + 400 + 300)$ cal

$= 712.5$ cal

$= 712.5 \times 4.2$ J

Q.1 A body weights 40kgwt on the surface of earth, what will be it's weight on the surface of Mars of radius $1/3$ & Mass $1/2$ that of earth?

Q.2 Radio cyclone broadcasts at 25m. what is the frequency of the station (Given speed of radio wave $= 3 \times 10^8 m/s$).

Q.3 Temperature of certain bath given the same reading on both centigrade & Farhrenhite scale. calculate the temp. of the bath.

Q.4 A piece of copper wire has a length of 10m. at $0°c$. Find its length at $100°c$. (Given $\gamma = 51 \times 10^{-6} °c^{-1}$)

Q.5 How much gram of water 20°c will evolve 1 Kcal of heat by becoming ice at 0°c.

Q.6 50g of water at 0°c is mixed with an equal mass of water at 53c. Find the equilibrium temp of the mixture.